MATH 54 - HINTS TO HOMEWORK 2

PEYAM TABRIZIAN

Here are a couple of hints to Homework 2! Enjoy :)

Section 1.7: Linear independence

1.7.1. The easiest way to do this is to form the matrix whose columns are the given vectors and see if there is a pivot in each row!

1.7.3. Notice the second vector is -3 times the first vector!

1.7.15, 1.7.17. A set with more than 2 elements in $\mathbb{R}^2$ is always linearly dependent. A set with the zero-vector is always linearly dependent.

1.7.21.

(a) **F** (the equation $A\mathbf{x} = \mathbf{0}$ **always** has the trivial solution, no matter what the columns of A look like!)
(b) **F** (for example, $S = \left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} \right\}$ doesn't satisfy this! The correct statement should be: there is **some** vector such that $\cdots$)
(c) **T** (in other words, 5 vectors in $\mathbb{R}^4$ are linearly dependent)
(d) **T** (otherwise the set would be linearly independent)

1.7.33, 1.7.34, 1.7.37. Remember that a set is linearly dependent if there's a relationship between the vectors in the set. Also, a set with the zero vector is always linearly dependent.

1.7.36. False (choose $\mathbf{v_1} = \mathbf{v_2} = \mathbf{v_4}$ and $\mathbf{v_3}$ linearly independent from $\mathbf{v_1}$! The point is for linear independence, you have to consider the set as a whole!)

1.7.37. True

Section 2.1: Matrix operations

2.1.7. Remember the rule $(m \times n) \bullet (n \times p) = (m \times p)$.

2.1.9. Write out AB and BA and solve for k in $AB = BA$.

2.1.13. $[Q\mathbf{r_1} \cdots Q\mathbf{r_p}] = Q\,[\mathbf{r_1} \cdots \mathbf{r_p}]$. Think of this in terms of how you multiply matrices! **Remember this for later on, it will be useful!**

Date: Tuesday, June 26th, 2012.

2.1.15.

(a) **F** (oh, life would be awesome if this was true! But $\mathbf{a_1 a_2}$ doesn't even make sense!)
(b) **F** (the columns of **A** using weights from the column of **B**)
(c) **T**
(d) **T**
(e) **F** (in the *reverse* order, $(AB)^T = B^T A^T$)

2.1.16.

(a) **F** (that doesn't even make sense because AB has to be a 3×3 matrix! The $+$ shouldn't be there!)
(b) **F** (column, not row)
(c) **F** (in general $BC \neq CB$)
(d) **F** (it's $B^T A^T$)
(e) **T** (that's just a lengthy way of saying $(A+B)^T = A^T + B^T$)

2.1.23. Multiply the equation $A\mathbf{x} = \mathbf{0}$ by C.

2.1.25. On the one hand, $CAD = (CA)D = ID = D$, on the other hand $CAD = C(AD) = CI = C$, hence $C = D$

SECTION 2.2: THE INVERSE OF A MATRIX

2.2.1. Use theorem 4.

2.2.5. Use the fact that if A is invertible, and $A\mathbf{x} = \mathbf{b}$, then $\mathbf{x} = A^{-1}\mathbf{b}$.

2.2.9. All statements are true, except for (b), because $(AB)^{-1} = B^{-1}A^{-1}$.

2.2.10.

(a) **F** (*reverse* order, $(AB)^{-1} = B^{-1}A^{-1}$)
(b) **T** ($\left(A^{-1}\right)^{-1} = A$)
(c) **T** (because then $ad - bc = 0$)
(d) **T** (this follows from the Invertible Matrix Theorem, see next section)
(e) **F** (reduce I to A^{-1})

2.2.13. Muliply the equation $AB = AC$ on the left by A^{-1}. Not true in general, consider for example A to be $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ and B and C any two matrices!

2.2.18. $B = P^{-1}AP$

SECTION 2.3: CHARACTERIZATIONS OF INVERTIBLE MATRICES

2.3.3, 2.3.5. Row-reduction is the key!

2.3.11, 2.3.12, 2.3.18, 2.3.22, 2.3.24. Just look at theorem 8! If one of those statements holds, then all of them hold!